
Marcoux is BAE
Introduction to Analysis
Section 0. Induction
0.1: The Well-Ordering Principle
0.2: First Principle of Mathematical Induction
0.3: Second Principle of Mathematical Induction
Section 1. The Real Numbers
1.1: Supremum and Infimum
1.2: LUB/GLB Property
1.3: The Archimedean Property of N
Section 2. Relation, Functions, and the Uncountability of R
2.1: Cartesian Product, Relation, Function, Domain, Codomain, Range
2.2: Injection, Surjection, Bijection, Function Composition
2.3: Cardinality
2.4: TFAE on Cardinality
2.5: Cartesian Product on Countable Sets is Countable
2.6: The Uncountability of R
The Lmi of Sequences
Section 3. Sequences and Limits
3.1: Sequences, Bounded Sequences, The Limit of a Sequence
3.2: Addition and Scalar Multiplication of Limits
3.3: Multiplication of Limits
3.5: Subsequence
3.6: A convergent sequence is bounded
3.7: Subsequences of (Xn) converge to the same limit as (Xn)
3.8: The Squeeze Theorem
3.9: Extended R and Properly Divergent Sequence
3.10: The Comparison Theorem
Section 4. Bounded Sequences
4.1 Monotone Convergence Theorem
4.2 Bolzano-Weiserstrass Theorem and Nested Interval Theorem
Section 5. Cauchy Sequence
5.1 Cauchy Sequence and Convergence implies Cauchy
5.2 A Cauchy sequence is a convergent sequence.
5.3 Sequentially Compact, [a,b] is sequentially compact.
Section 6. Limit Superior and Limit Inferior
6.1: Definition of Lim sup and Lim inf (with unbounded sequences)
6.2: Alternative definition and theorem; the infamous theorem 6.6
6.3: For a convergent sequence, lim inf = lim = lim sup
6.4: Remark
Additional Topics
Section 13. Taylor's Theorem
13.1: Definition of Taylor's polynomial and Lagrange Remainder term
13.2: Taylor's Theorem (Proof not needed)
13.3: e is irrational (Proof not needed)
Section 14. L'Hopital's Rule
14.1 Theorem V1 (Proof not needed)
14.2 Theorem V2 (Assume V1, prove V2)
14.3 Theorem V3 (Proof not needed)
Section 15. Inverse Functions
15.1 Function and its inverse
15.2 Inverse is a function iff f is bijective
15.3 Continuity and injection implies monotonicity of f
15.4 Continuity and injection implies the image is an interval and the inverse is continuous
15.5 Derivative of inverse function
Limits of Functions
Section 7. Limits
7.1: Definition of punctured neighborhood, limit of a function, and basic operations of limits
7.2: Sequential characterization of limits
7.3: Operations on limits with sequence-based proofs
7.4: The Squeeze Theorem
7.5: The Comparison Theorem
Section 8. Limits II
8.1: f(x) converges to b as x approaches y from the left/right
8.2: f(x) diverges to positive/negative infinity as x approaches y from the left/right
8.3: f(x) converges to b as x tends to positive/negative infinity
8.4: f(x) diverges to positive/negative infinity as x tends to positive/negative infinity
8.5: One-sided limit vs. Two-sided limit
Section 9. Continuity
9.1: Definition of Continuity
9.2: Sequential Characterization of Continuity
9.3: Operations involving continuity
9.4: Theorem on function composition
9.5: Bounded function
9.6: The Extreme Value Theorem
9.7: The Intermediate Value Theorem (Proof not needed)
9.8: Different forms of discontinuities
Section 10. Uniform Continuity
10.1: Uniform Continuity
10.2: (Xn) Cauchy implies (f(Xn)) Cauchy
10.3: Continuity is equivalent to uniformly continuity
Section 11. Differentiability I
11.1: Definition of Derivative
11.2: TFAE on Derivative
11.3: Differentiability implies continuity
11.4: Operations on derivatives
11.5: Derivative of polynomials
11.6: Local maximum/minimum and proposition
11.7: The quotient rule
11.8: The chain rule
Section 12. Differentiability II
12.1: Rolle's theorem
12.2: Mean value theorem (Proof not needed)
12.3: Cauchy's mean value theorem
12.4: Differentiability + bound derivative imply uniform continuity
12.5: Derivative vs. Graphical Representation of f
12.6: Function vs. One-sided derivative
12.7: Countably-many discontinuities
12.8: Darboux's theorem
12.9: Concavity (Proof not needed)
12.10: Point of inflection and Proposition
12.11: Graphing
Assignments
Assignment 7
Open and Closed Sets in R
7.1: Definition of open and closed sets; empty set and R are both open
7.2: The union of an arbitrary collection of open sets in R is open
7.3: The intersection of a finite collection of open sets is open
7.4: The intersection of a finite collection of closed sets is closed
7.5: An open interval is open; a closed interval is closed
7.6: TFAE on closed sets
Sequential Compactness
7.7: A subset of R is sequentially compact iff it is closed and bounded
7.8: The image of a continuous function from a sequentially compact set is bounded
Assignment 8
8.1:Continuity on a set
8.2: TFAE on continuity on a set
8.3: Uniform continuity on a set
8.4: Example on uniform continuity
8.5: Example on discontinuity
8.6: The continuous image of a sequentially compact set
Assignment 9
9.1: Discontinuities for increasing functions
9.2: Examples on derivatives
9.3: Local and global extrema
Assignment 10
10.1: Curve Sketching
10.2: Differentiability
10.3: L'Hopital's Rule
10.4: Taylor Polynomials
10.5: Inverse Functions